Shaving tool designing

The shaving cutter (or shaver) is, substantially, a gear that mates with crossed axes with the wheel to be shaved.
Due to the non-parallelism of the axes a reciprocal sliding is generated between the active surfaces of gear and shaver.
Since a number of serrations are slotted along teeth flanks, their edges becomes cutting portions giving to the sliding action the power to remove chipping and therefore to finish gear teeth flanks.
Shaving operation is an accurate process, the gear must have profiles and leads within a few microns; in order to obtain good results you do not require only efficient machines and fixtures, but you also have to design an make the shaving cutter according to appropriate criteria taking into account a series of statical and dynamical phenomena occurring during the process.

The Even Contacts method
The most widely used shaving cutter designing system is the so called *Even Contacts Method*.
This method consists in developing a shaver that during its mating with the workpiece always has an even number of points in contact between shaver and workpiece teeth flanks.
The conditions outlined in figure N°1a is not easy to achieve and, above all, it is not easy to maintain both during all the phases of the rotation and throughout the shaver life.

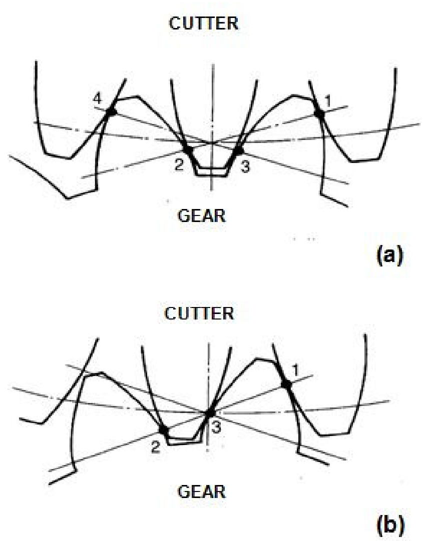


Fig.N°1- *Indication of the number of point of contact between gear and cutter*
(a) – Even contact
(b) – Odd contact

Far too often, work piece geometrical characteristics and the other surrounding conditions can be an obstacle to achieve such feature.
The restrictions that we have to take into account are, for instance, the minimum and maximum centre distance that the shaving machine can accept, the number of teeth of the shavers that should be prime against the workpiece number of teeth, the Start of Active Profile (SAP) of the gear, etc.
Why is the even contacts situation so desirable?

The answer is simple and, after all, fairly intuitive: it is important to balance the forces involved.
If you go and consider figure N°2a, you understand that force F that pushes the cutter against the gear can be divided in 2 main components: one pushing on RH flanks and the other pushing on LH ones.

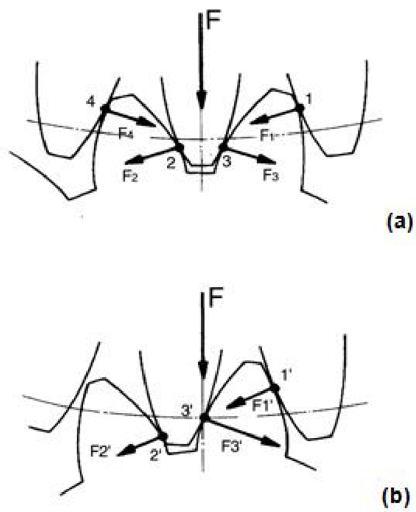


Fig.N°2- *Indication of the forces acting on each single point of contact*

In the a.m. instance, each of the two components is divided in 2 forces that operates always along the same line of action and that have an equal intensity.
Forces F_1, F_2, F_3, F_4 acts then on point 1, 2, 3, 4 in the same way and the cutting action in these points will always be the same; if such condition is constantly kept, then you can expect that gear profile comes out regular, as it happens in most of the cases.
But if in a determined moment of the rotation the number of points in contact is odd, like in figure 1b, then the situation can be schematised as in figure 2b.
You can see that the module of vector $F_{3'}$ is equal to the sum of vectors $F_{1'} + F_{2'}$, and in the point 3' the cutting action will be different with reference to the situation of point 3.
In other words, the cutter pushes against point 3' with a force double than the one exerted on points 3 and 4.
It is evident that in point 3 the metal removal will be higher and therefore you are going to have a profile irregularity in that point.
The main reason, anyway, because you have to avoid the odd contacts situation is that you run the risk of a gear tooth distortion, due to the unbalanced forces that would tend to bend the tooth.
It sounds impossible that gear tooth might bend, but you cannot forget that forces involved are quite high and that the expected variations of tooth aspect are considered of some microns.
If you look at figure N°2b you can see, first of all, that on point 2' it is acting $F_{2'}$, the only one pushing against the tooth flank, thus contributing to bend it.
But of point 3' the force $F_{3'}$ is considerably bigger than $F_{1'}$ and also in this case forces are unbalanced with the risk of bending the tooth.

The effect of variation of the unbalanced forces is experientially proven, even if things are not always going on in the same way, as we will see at a later stage.
If the even contacts condition is lost, in the majority of cases the gear profile you obtain is more or less as shown in figure N°3.

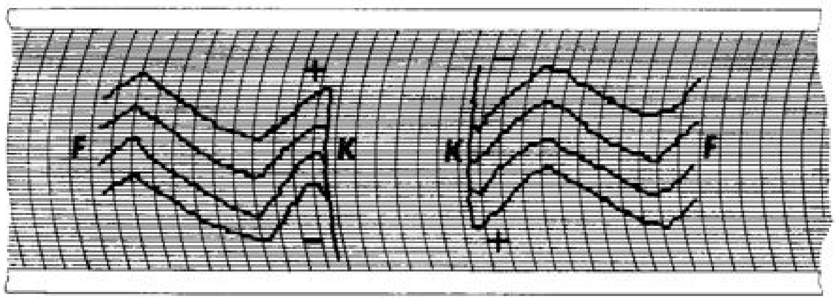
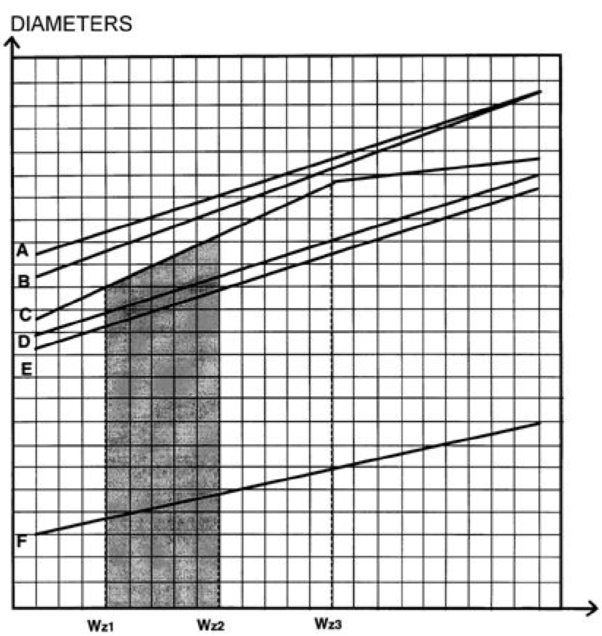


Fig.N°3- *Typical profile error generated by the non fulfilment of even contacts condition*

In the central area you can see that metal removal is bigger than required.
The designing of a shaving cutter is nowadays developed with the aid of computers using sophisticated softwares, but the designer interventions always necessary.
In fact the computer can supply a series of informations and a series of options for the designer to chose, thus deciding how the tools will be made.
Among the various informations given our by the computer there is special diagram shown in figure N°4, which shows what happens to the cutter during the various resharpenings. This diagram puts into relationship the tooth thickness (abscise axis) with the various diameter of the tool.



A = Root diameter
B = Beginning of root fillet
C = Even contacts
D \ E = Start of active profile (SAP)
F = Outside diameter of gear
Wz = Thickness over Z teeth

Fig.N°4- *User diagram of shaving cutter*

For instance, line **C** indicates how the shaving cutter outside diameter must be in function of the tooth thickness, in order to maintain the even contacts condition. Such diagram is particularly important because it must be always available when resharpening a shaving cutter. In fact, the more tooth chordal thickness reduces due to resharpening, the more cutter outside diameter must be reduced, according to the line **C**. *This is also called the resharpening diagram.*
If you made a cutter with diameters corresponding to line **A** then the cutter would go and touch the gear root and it could not work properly. The same would happen if the diameters are chosen corresponding to line **B**: the cutter would go and touch the beginning of root fillet, with negative influences on the profile.
Lines **D** and **E** indicates the cutter diameters that would touch the gear profile points placed in correspondence with start of active profile (SAP) in two different options.
Line **F** indicates the cutter diameter that touches the profile point placed in correspondence of outside diameter or start of chamfer diameter. The **W** value, surveyed on several teeth, is the equivalent of the tooth chordal thickness. Its possible variation has a quite wide range, surely wider than the sum of serration depth. This means that designer must chose which graph sector he has to consider for the cutter.
There are some elements that are quite important for the choice of graph sector.
For instance, in figure N°4 graph you can see that in correspondence of $\mathbf{W_{z3}}$, the cutter outside diameter that would guarantee the even contacts condition is quite close to the diameter that would touch the beginning of root fillet, this being a fairly dangerous situation, that is better to avoid.

It is better to act as shown in the same diagram, i.e. to choose a use area included between values $\mathbf{W_{z1}}$ and $\mathbf{W_{z2}}$, where there no danger of interference, and where it is surely guaranteed the shaving of the whole working length of the tooth.
Another important feature to be taken into consideration is shaving cutter tip land. The tool use area, wherever possible, should be chosen in a way that allows tooth tip land to be equal or bigger than the total depth of the serrations on the two flanks. If this conditions is not coped with on tooth tip area, then you generate serration with a quite small section, easily subject to breaking.
Generally speaking, if you chose the condition that enables to have a big tip land, you generate a stub, but more resistant to flexion tooth.
There are no workshop results that confirm that a tool of such shape is performing better than a cutter with a slender tooth, i.e. slimmer at top; anyway you have to consider that the Japanese manufacturer, traditionally follow such path.

__The Even Contacts method limitation__
The a.m. shaving cutter designing method, as we said, is used by the majority of shaving tools manufacturers, but sometimes results are not as expected.
This means that such method cannot be used universally.
Let's make a few considerations:
1. *The component forces making up **F** force always act along line of action. This means that they are always inclined of an angle equal to operating pressure angle.*
2. *These forces always act perpendicularly to the contact surface.*
3. *Forces pushing on the flanks are subject to an inverse proportion with reference to the operating pressure angle. If you think in terms of vectors modules, you can write* $F_1 + F_2 + F_3 + F_4 = F: 2\ sen\alpha$.

4. In reality, there are no points of contact, but contacts areas more or less wide. It is obvious that when 2 curve surfaces are pushed one against another, the contact is not punctiform, but because of the effect of steel plastic deformation and of the cutting action, and also of penetration, you will obtain a more or less wide contact area.

5. The previous schemes refers to a radial section of the teeth. In reality the contact happens along an oblique path of variable length and its inclination depends on mating characteristics (see figure N°5a). In the case of plunge type shavers with hollow helix, the contact area can cover the whole tooth length (see figure N°5b).

The width of contact area, which is more or less shaped as an ellipse, depends on many factors and there are also many formulae that enable to calculate the small axes ($2b_e$) and the big axes l_e of such ellipse.

For instance, in agreement with **W.A.Tuplin** (*Crossed Axes Shaving of Gear Teeth* in *The Machinist 10/09/49)* or **G.Henriot** *(Gears . A Practical And Theoretical Essay Ed.Tecniche Nuove 1978 - volume 2)* and **Huette** (*The Engineer Handbook - volume 1)* you can have the following formulae :

$$l_e = \frac{2}{sen(\beta_1 \pm \beta_2)} \cdot \sqrt{\frac{2 \cdot b_e \cdot (R_1 \cdot \cos^2 \beta_2 + R_2 \cdot \cos^2 \beta_1)}{sen\,\alpha_{on}}}$$

$$b_e = 9{,}56 \cdot \sqrt{\frac{F_p \cdot \rho}{E \cdot l_e}} \qquad \text{where} \qquad \frac{1}{\rho} = \frac{1}{\rho_1} + \frac{1}{\rho_2}$$

and where the numerical coefficient have been grouped.
The value of b_e that allows an appropriate cutting action is approximately 2÷3 microns.
You do not have to forget that in such formula an important feature has not been taken into account: the shaving cutter has got serrations and therefore the contact area of the two surfaces is reduced to approximately an half, and the value of F must be appropriately increased.
In practice, contact area is striped, as indicated in figure N°5c, and the enveloping of these partial areas is an ellipse with bigger dimensions than the one you would have between two normal gears.
The length of the flattening area l_e depends very much on the value of cross-of-axes angle. Low values of γ gives low values of l_e while as the value of cross-of-axes angle increases, the length of the flattening area rapidly increases accordingly.
The increasing of l_e value has as a direct consequence the increasing of the contact area. And therefore a decreasing of the pressure in contact area. This makes cutting action more difficult.
The value of l_e increases also in function of the increasing of the diameter of the gear to be shaved. This is the reason why in the shaving of large gears the cross-of-axes angle has noticeable values : even 45°. Only in this way you can keep to the l_e value, and therefore in the range of force applied, within acceptable limits.
It can be of certain importance to notice that contact area is inclined of a certain angle μ with reference to the generating line and, in agreement with W.A.Tuplin, we have :

$$tg\mu \approx tg\frac{\beta_1 - \beta_2}{2} \cdot sen\alpha_{on} \approx tg\frac{\gamma}{2} \cdot sen\alpha_{on}$$

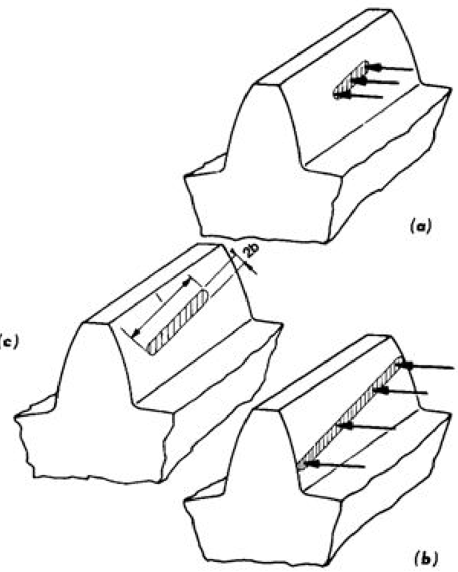


Fig.N°5- *Schematics of the contact area width*

Having said the above, all we still need to say about the points in contact is that they will hardly be kept constant, i.e. if the initial condition is of 4 points of contact, during the rolling, in any moment they can became 2 or 6, but hardly ever they will constantly remain 4.
The even contacts condition is respected, but not the one of the constancy of the number of points of contact.
This can cause some doubts about the general validity of such system.
In fact, if you consider the radial force F, when you, for instance, pass from the 4 points condition to the 2 point one, the force acting on every point is double and then in these determined points the metal removal will be higher.
It is true that you have balanced forces, which avoids the eventual flexion of the tooth, but you have you admit that you find a noticeable variation of the intensity of the forces acting on the teeth and, then, expect some negative outcome.
The length of line of action plays an important role because the number of points of contact depends on this element.
If the line of contact is long and the points of contact have a sequence 4-6-4 on each flank you can have 2 or 3 points and therefore the variation of the force involved is approx. 30%.
Instead, if you pass from 2 points of contact on each flank to 1 the variation of forces would be 50%.
In the first instance, profile would result more constant than in the second case.
This is in agreement with workshop practice which has shown that it is easier to shave well gears with a high number of teeth, while for low number of teeth e.g. 12÷14 you nearly always find problems in achieving the required profile.
With a high number of teeth on workpiece it is easier to obtain a contacts condition 4-6-4, while this is nearly impossible if you shave gears with low number of teeth.

Another reason that makes even contacts low a little aleatory, is that it is based of the balance of forces and on the concept that if you have equal forces, then you have equal cutting action.
This is not true in absolute terms ; it would be more correct to state that if you have an equal pressure, i.e. specific force applied, then you have an equal cutting action.
But pressures are not equal during the shaving process, even if forces remain constant.
We have stated that in every single point the contact between gear and cutter takes place in an area that is as bigger as higher is the force applied and as bigger is the curvature of the two surfaces in contact.
We remind that curvature ρ is the inverse of the curvature radius, i.e. the smaller the radius, the bigger the curvature.
The shaving cutter as a pitch diameter that normally measures 200 mm and then the gear flanks have a fairly constant curvature, which cannot be true for the gears with low number of teeth.
Close to base diameter the curvature is high, close to outside diameter the curvature is much smaller: the consequence is that also contact areas and pressures in contact area would become quite different.
We can then state that material removal close to root diameter, provided applied forces are equal, will be bigger than those taking place close to outside diameter.
This reasoning still does not follow a precise line, as the problem is complicated by other elements that have not yet been taken into consideration.
The first is that if we do not consider pitch diameter, in every point you have a sliding of cutter and gear surfaces in a radial direction (i.e. from root to top, or vice-versa); such sliding is as more pronounced as farther we go from operative pitch diameter .
Which is the effect of this sliding on the width of the contacts area and on the cutting action ?
The second element to be considered is that because of cross-of-axes angle there is a longitudinal sliding, along the helix direction.
It is such sliding that generates the real cutting action .
Also about this item would require a deeper analysis of the chipping generation, in particular about the interaction between the width of contacts area, its position and the cutting direction which is continuously variable.
During the designing of shaving cutters the above features are normally not taken into consideration, same as another force that surely has a certain influence on the size of material removal.
This force is the one that generates the rotation of the gear-cutter system.
This is a force that, especially for gears of a certain size, is not negligible and the very same inertia of the system avoids that short unbalances can have a great influence on the accuracy of the operation.
Also the force exerted by the cutter against the gear is quite big and it is clear why its eventual strong and prolonged unbalance can cause noticeable differences in the tooth bending and then on the gear profile obtained.
SICMAT S.p.A. Turin (a company making shaving machines, subject we will touch at a later stage) gave out data about radial force F to be approx of 400 Kg for a cutter working on module 2 mm, 20° pressure angle and face width 20 mm.
Considering a 4 points of contact condition, in each single area you will have a force of approx.
$F_1 = F_2 = F_3 = F_4 = F : 4 \ \mathrm{sen}\,\alpha = 400 : 4\,\mathrm{sen}\,20° = 292$ Kg
If during the rotation you pass from 4 to 2 points of contact you have the doubling of this force.

You can now have an idea of why do not always run the way we would like.
Instead, if you pass from a 4 points of contact condition to 6 the force on each single area of contact would be 196 Kg; the effect is not always noticeable.
In the end, we should consider also the cutter-workpiece group inertia.
During shaving loads can very quickly, just a few microseconds to go from one situation to the other.
The shaving cutter and its clamping spindle have a noticeable inertia. The same happens for the workpiece and its clamping. It is not too easy to quickly vary their dynamic condition. Which is the role played by inertia in the system?
By intuition, it should play a positive effect, but its quantity evaluation is another open issue.
From these short and somehow rough notes, you can understand that shaving is a very complex process; variables playing roles are many and of difficult interpretation. The result, fairly often easy to forecast, sometimes is not satisfactory.
In these instances, the designer or the analyst have nothing to back them up, groping in the dark or in the thick fog, and have to look for assistance in the past and long experience, previous case studies, trials, and so on.
As a conclusion of this chapter, we would not like that somebody having a well working shaving cutter would think he is particularly happy: the vast majority of cutters is working well since its first use and continues to perform satisfactorily provided it is correctly resharpened.
The problems are normally referred to borderline cases and it in these circumstances that user must realise the existing difficulties and offer its co-operation in order to improve the results.
As you will see later on, when we go through some examples of shaving cutter designing, it is not easy to always cope with even contacts conditions .
Shaving cutter outside diameters are normally determined considering other restrictions and, notwithstanding this feature, the results are nearly always acceptable.

Designing according to even contacts law

- *Guarantees forces balance*
- *Avoids workpiece tooth bending*
- *Does not guarantee a uniform stock removal removing*
- *In case of gears having a low number of teeth, even strictly coping with this method, results are not positive in many instances.*
- *In many instances you cannot strictly apply this method, but yet, you can obtain acceptable results*
- *Protuberance relief disturbs the strict application of this method*